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Pricing Brazilian ABS to Trade in the Secondary Market: *Benefits from Assimilating Best Pricing Practices*

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Brazilian asset-backed securities (ABS) appeal to investors because of their relatively high yields and diversification benefits. However, Brazilian and international investors struggle to price these ABS because of the country's unique financial calculations, lack of transparency, and the scarcity of quantitative pricing tools in Brazil. All market participants would benefit from the increased liquidity and capital flows that would result from adopting the best practices for pricing that we describe in this article. To this end, we also propose norms for standardizing the information needed to trade in secondary markets for Fundos de Investimento em Direitos Creditórios (FIDCs, the most prevalent ABS vehicle).

The benefits would stem primarily from improving the secondary market for FIDCs, which is probably the most illiquid of Brazil's relatively illiquid secondary fixed-income markets. FIDCs trade infrequently and with huge bid–offer spreads. The vast majority of FIDC investors hold FIDCs until maturity. This could change for the better if the market had the tools and the data to make informed decisions.

PRICING CHALLENGES OF BRAZILIAN FIDCs

Financial calculations in Brazil differ significantly from almost every other country in the world. Although Brazil's unique calculations have their advantages, they discourage the assimilation of financial technology from developed markets. This is because software developers typically don't find it financially rewarding to adapt software to Brazil's many rules and idiosyncratic calculations. Although a few local banks have developed technology to price FIDCs, they do not share their approach because they view it as "top secret." This is unfortunate, because everyone could benefit if the entire market adopted the technology, which would improve information for trading and thereby improve liquidity in the secondary markets.

The Brazilian market lacks transparency, standardization, and rapid access to the information needed to calculate prices quickly and correctly. For example, administrators hold all of the historic data and disseminate the information as accounting numbers rather than in trader-ready formats. Dissemination is slow. The data are published on a monthly basis with a 15-day delay on the Comissão de Valores Mobiliários (CVM) and Central de Custódia e Liquidação Financeira de Títulos (CETIP) websites. The data are difficult to download and are so disorganized that even Bloomberg treats FIDCs as investment funds instead of asset-backed securities. Uqbar's Orbis is the only service that provides aggregation and easy access to data for Brazilian ABS, but it does not have the services to provide pricing information to a pricing application.

The slow, trader-unfriendly data dissemination isn't the only problem. The FIDC prices from the administrator combine original face, accrued interest, and the amortization factor for many FIDCs. This frustrates investors, who frequently need a complete history of the interest rates and holidays over the life of the deal to obtain the data for calculating a market price. Formulas for calculating coupons vary from deal to deal, and seemingly simple things like rounding conventions are not standardized. Even coupon and amortization calculations can vary. We show how to handle some of these differences to permit the use of more advanced pricing applications.

ADVANTAGES OF USING QUANTITATIVE PRICING METHODOLOGY

We explain in this article why simple spread and yield-to-maturity calculations are not adequate to handle Brazil's wide variety of payoff profiles—soft bullets, straight-line amortization, mortgage-style amortization, to name a few. Developed markets solved many of these problems years ago. However, calculation conventions for the Brazilian market have complicated the implementation of those solutions in Brazil. The business day/252 day count is probably the most infamous of these conventions. Then there is the daily compounding interest formula for Brazil, which is sometimes calculated as a spread added to the Certificado de Deposito Interbancário rate (CDI or DI—Brazil's equivalent of LIBOR) or as a multiplicative spread expressed as a percentage of the CDI rate. These are just a few of the differences in market conventions.

Brokers/dealers do not provide deep and complete markets because investors hold to maturity, since they generally do not have the information or tools to trade. Most sales are forced as a result of an investor request for cash or a violation of investor guidelines. This could change if Brazilians follow our suggestions on how to combine CVM and CETIP data with leading-edge pricing technology to consistently price FIDCs for all amortization and coupon-calculation methodologies.

This article does not introduce any new financial technology but modifies current best practices for use in pricing Brazilian ABS. The text is basic because the idea is to explain best developed-market practices to Brazilian investors and Brazilian market conventions to foreign investors. The article first discusses the different types of FIDCs and then provides instructions on how to calculate coupons for the two most prevalent coupon formulas. We show how the investor can use information provided in reports published by the CVM to calculate components of the prices, such as accrued interest and consistent spreads. Finally, we price and analyze two FIDCs using best practices for pricing. This article aims to remove at least a few of the road blocks to market liquidity that stem from lack of pricing technology.

OPEN FIDC VERSUS CLOSED FIDC

Legally, a FIDC is an investment fund that is restricted to purchasing private credit, which is the debt of private companies or individuals. Per regulations from the CVM, FIDC operators must invest at least 50% of the fund in private credit and the balance must be invested in liquid government securities or high-grade, low-risk investment funds. As in the fund industry for most developed markets, FIDCs are offered in an open or closed format. (For a more complete description of FIDCs, see Budinger [2012]).

Pricing Considerations—Redemption for Open versus Amortization for Closed

Open FIDCs can be redeemed while closed FIDCs are amortized, and this drives their pricing formulas. Open FIDCs generally accumulate interest until the shares are redeemed, when the interest is paid with the principal in the manner of a Z-bond (PIK bonds are often used as an example, but this is not quite accurate because PIK bonds are often paid off with other securities). Amortizations of closed FIDCs are actually redemptions implemented for all shareholders on set dates. Exceptions may occur if the terms of the bylaws have been violated or a shareholder disagreement allows one investor to pull out. As we shall see later on, amortization of a closed FIDC reduces the cota value or price; however, redemptions by individual investors do not affect the cota value of an open FIDC. To summarize, the open FIDC's price continues to accrue, but the number of cotas change with redemptions. A closed FIDC's price drops when there is an amortization, but the number of cotas outstanding stays the same.

A cota is conceptually the same as a tranche. The English translation for cota is quota, sometimes used for mutual funds. We decided to use cota for this piece because this does not require translation of many of the terms that we refer to in government reports. However, there is one key difference between a tranche and a cota: A tranche has legally binding obligations to pay interest whereas the coupon for a cota is stated as a target return. Compliance triggers generally take effect and redirect the waterfall before the coupon payment falls below the stated target return.

FIDC prices quoted in the CVM reports are book entries with accrued interest and are loosely tied to the net present value of future cash flows. There are two peculiarities of note for foreign investors. First, FIDCs resemble investment funds in the United States or other markets in that the *valor da cota* (cota value) establishes the par or face value and that the number of cotas times the per-share value of cotas establishes the total value issued. FIDC cota values and units resemble prices for stocks and closed-end funds more than bonds. Because the cota value is different for each cota, there is no consistent meaning for 100% of par as the price is used in developed markets. This is another factor in pricing that should be standardized to fit with the rest of the world.

The historic coupon return of an open FIDC can be easily calculated using the cota value from the Informe Mensal (CVM Monthly Report).

Generally, the cota value for an open FIDC is

$$Value_{today} = Value_{t-1} \times FactorDI$$

where

$Value_{t-1}$ = accumulated value of the equation, except on the issue date when it equals the issue value of one cota

$$FactorDI = [(\text{CETIP factor} - 1) \times \text{Spread multiplier}] + 1$$

CETIP Factor = 1 + one day's accrued CDI interest on one Brazilian real based on a business day/252 day-count, calculated by CETIP.

For example, an open FIDC issued on June 29, 2012, with a cota value of R$10,000 and earning 120% of CDI (8.36%) would be valued on July 2, 2012, using the follow calculations:

$$FactorDI = [(1.00031939 - 1) \times 1.20] + 1 = 1.000383$$
$$\text{Value}_{\text{July 2, 2012}} = \text{R\$}10{,}000 \times 1.000383 = \text{R\$}10{,}003.83$$

So, the annualized lifetime realized coupon return on an open FIDC is

$$RCR = (CV_t/CV_0)^{(252/HP)} - 1$$

where

HP = holding period in business days
CV_t = cota value on day t

Note that investors should also consider that open FIDCs are subject to a tax schedule that starts at 22.5% and declines to 15% after two years. This tax is implemented through the "come cotas" tax ("eat the cotas" tax); that is the administrator is forced to redeem cotas to pay the tax. Closed FIDCs are subject to the same tax schedule if the coupon payments begin before 180 days. Closed FIDCs that start to amortize after two years (which is the Brazil definition of long term) are subject to a flat 15% tax, which is why we often see the amortization and payment of interest start after two years on closed FIDCs. Closed FIDCs do not have forced redemptions to pay tax liabilities. The tax is paid only if there has been an interest payment, and this provides cash to pay the tax.

Closed FIDCs are more complicated to price than open FIDCs, especially when they begin to amortize. This is because the cota value includes the original cota value at issue, the amortization factor, and the accrued interest all in one number.

$$Cota\ Value_{Closed\ FIDC(t)} = (Cota\ Value_{issue} \times \text{Amortization factor}) + \text{Accrued interest}$$

The amortization factor must be calculated and equals current amortized portion of the issue amount divided by the original amount issued.

$$\text{Amortization factor} = (Cota\ Value_{Closed\ FIDC(t)} - \text{Accrued interest}) / Cota\ Value_{issue}$$

where

$$Cota\ Value_{Closed\ FIDC(t)} = \text{Cota value for the current day}$$

Also, the investor should note that interest accrual is not based on the $Cota\ Value_{issue}$ but $Cota\ Value_{Closed\ FIDC(t)}$, which includes all past accrued interest. This is true for virtually every FIDC that we have seen and evolves from its legal structure as a mutual fund. The accrued interest component can grow to 30% or more of the cota value as seen in this article (Exhibit 7).

COUPON CALCULATIONS, BOOK VALUE PRICING, AND AMORTIZATION FOR CLOSED FIDCs

We chose two FIDCs, shown in Exhibit 1, to demonstrate how pricing in the market compares among FIDCs with the different coupon calculations and amortization formulas in the market. Chemical V is a FIDC from the Chemical program for Braskem, the giant Brazilian petrochemical firm. Braskem uses the Chemical FIDCs to help finance its receivables. Omni Veículos V (Omni V) was issued by Omni Financeira, a small Brazilian retail credit operation that specializes in financing used cars.

Although both FIDCs have a revolving investment portfolio, the two cotas are quite different and provide excellent examples of the different styles of amortization and coupon calculations in the Brazilian FIDC market.

Spread-Plus CDI Coupon Formula

The following formula for calculating the Cota Value was published in the prospectus for Chemical V:

$$VQS_T = (VQS_{T\text{-}1} - VAE_{T\text{-}1} - VAP_{T\text{-}1}) \times [(1 + (DI_{T\text{-}1}/100) \times (1 + Spread_{Sn}/100)]^{(1/252)}$$

where

VQS_T = unitary value of each senior cota calculated on day T

$VQS_{T\text{-}1}$ = unitary value of each senior cota calculated on the day immediately anterior to the data T. For the calculation on the first working day after the issue date, $VQS_{T\text{-}1}$ will be equal to price at issuance.

$VAE_{T\text{-}1}$ = unitary value of the extraordinary amortization

$VAP_{T\text{-}1}$ = unitary value of the periodic amortization effectively paid to the holders of each senior cota, on the working day immediately after day T

$DI_{T\text{-}1}$ = DI rate referring to the working day before day T

$Spread_{Sn}$ = spread factor of the senior cotas, expressed in annual percentage terms, 252 working days base.

Note that there were no rounding or truncating conventions specified in Chemical's pricing formulas.

The price for the first two days after issuance is calculated in Exhibit 2. If we continue this calculation,

EXHIBIT 1

Chemical V vs. Omni Veículos V

	Chemical V	Omni Financeira V Senior Series 2
Term	36 months	48 months
Amortization Style:		
Interest	Semi-annual until amortization, then monthly; no window exempted from interest payments	Start month 19, monthly thereafter
Principal	Soft bullet (last 6 months)	Start month 19, monthly thereafter
Spread	CDI + 1.25%	CDI × 112%
Rating	Aa1.br, Moody's	AA(bra), Fitch
Issue Size	272,400,000	145,496,596
Collateral	Accounts Receivable	Vehicle Loans
Comments	Similar to a regular bond with a soft bullet (principal paid the last six months)	Similar to a Z CMO where interest is paid in proportion to the principal paid
Issue Price	1,000 per quota	25,000 per quota

EXHIBIT 2
Basic Accrual Calculation for Chemical V

Date	Beginning Price	Historic CDI	One-Day Int. Factor	Ending Price
7/1/2010	1,000.000	10.12	1.000432	1,000.432
7/2/2010	1,000.432	10.14	1.000433	1,000.865

EXHIBIT 3
Chemical V Cota Pricing

Payment Date	CDI	Administrator Price	Price Based on Prospectus Formula
6/30/2010	10.12	1,000.00	1,000.00
7/30/2010	10.63	1,009.67	1,009.67
8/31/2010	10.61	1,019.72	1,019.72
9/30/2010	10.60	1,029.40	1,029.40
10/29/2010	10.64	1,038.72	1,038.72
11/30/2010	10.64	1,048.17	1,048.12
12/31/2010	10.64	1,000.45	1,000.45
1/31/2011	11.14	1,010.09	1,010.09
2/28/2011	11.16	1,019.60	1,019.60
3/31/2011	11.66	1,030.02	1,030.02
4/29/2011	11.89	1,039.62	1,039.62
5/31/2011	11.88	1,051.00	1,051.01
6/30/2011	12.12	1,000.00	1,000.00
7/29/2011	12.40	1,010.70	1,010.70
8/31/2011	12.39	1,022.70	1,022.70
9/30/2011	11.88	1,033.40	1,033.40
10/31/2011	11.39	1,043.00	1,043.55
11/30/2011	11.34	1,053.55	1,053.55
12/30/2011	10.87	1,000.00	1,000.00
1/31/2012	10.30	1,009.97	1,009.97
2/29/2012	10.28	1,018.41	1,018.41
3/30/2012	9.48	1,027.79	1,027.79
4/30/2012	8.73	1,036.03	1,036.03
5/31/2012	8.81	1,044.77	1,044.77
6/29/2012	8.36	1,000.00	1,000.00

Source: CETIP and CVM databases.

we replicate the complete pricing history of Chemical V until June 2012, the latest date available before this article was submitted for publishing (see Exhibit 3).

$$\text{Chemical V Cota Value}_{12/31/2010} = (\text{R\$}1{,}000 \times 1.0000) + \text{R\$}0.45$$

where

Original issue value = R$1,000
Amortization factor = 1.0000
Accrued interest = R$0.45

EXHIBIT 4
Chemical V Amortization Schedule

Month Payment	Interest	Principal (% of original value)
6	Yes	0%
12	Yes	0%
18	Yes	0%
24	Yes	0%
30	Yes	0%
31	Yes	16.66%
32	Yes	16.66%
33	Yes	16.66%
34	Yes	16.66%
35	Yes	16.66%
36	Yes	16.70%

Note that daily administrator prices for Chemical V are not available.

The Chemical V amortizes as a soft bullet and the amortization schedule is determined by a table in the prospectus. See Exhibit 4. The payoff schedule is shown in Exhibit 5.

Percentage of the DI Rate Pricing

The following percentage of DI pricing formula comes from the bylaws of Omni V:

$$Re = (VCse \times FactorDI)$$

Re = value accumulated for each cota for each period of capitalization. Calculated to six decimal places, truncated.

VCse = value of the Senior Cota before amortization at the beginning of the capitalization period. Calculated to six decimal places and then truncated.

FactorDI = the rate DI with the use of the factor multiplier, calculated from the initial date of the capitalization period, inclusive, until the date of the calculation, exclusive, calculated to eight decimal places rounded.

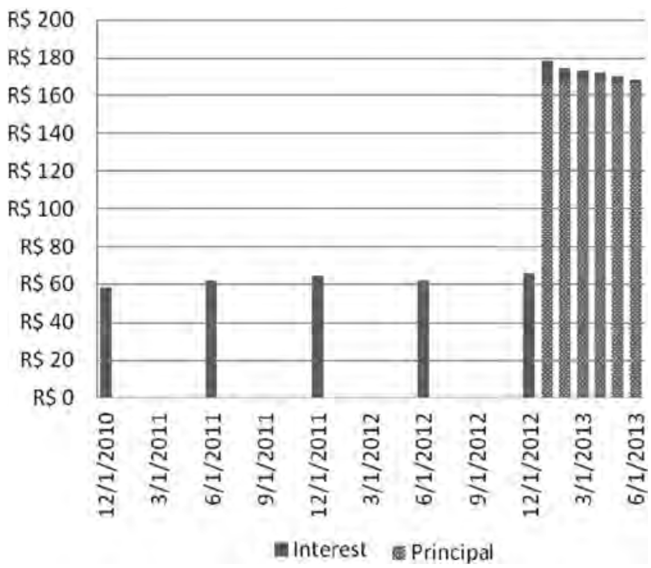
$$\textit{Factor}\text{DI} = (TDI \times FM)$$

where

TDI = DI rate over, expressed daily, published by CETIP

FM = factor multiplier, 112% as determined in the prospectus.

EXHIBIT 5
Chemical V Payoff Profile



We followed the calculations specified in the prospectus for both deals and compared them to the administrator cota values in the CVM reports. These cota values match during the entire history of the deal, although we only show one month for brevity. See Exhibit 6.

Amortization of Omni V:

$$PMT_{ij} = \{1/[T_i - (M_{i,j} - 1)]\} \times VQSn_{t,j}$$

where

- i is the series of the senior cotas
- j is the jth amortization
- T_i is the term or maturity in months for the ith cota
- PMT_{ij} is the jth amortization for cota i
- $M_{i,j}$ is the jth month of the life of the deal beginning after the issue date.
- $VQSn_{t,j}$ is the unitary value of cota

See Exhibit 7 for the Omni V price calculation with amortization and Exhibit 8 for the payoff profile.

$$\text{Omni Cota Value}_{12/31/2010} = (R\$25{,}000 \times 0.5000) + R\$4{,}655.83$$

where

Original issue value = R$25,000
Amortization factor = 0.5000
Accrued interest = R$4,655.83

We used older FIDCs because this allowed us to demonstrate the detailed calculations that replicate prices and to prove the model so as to eliminate any possible doubts about the cash flows driving pricing calculations. We also wanted to show the how difficult it is for traders to track the correct accrued interest and calculate the correct future coupon payments for pricing. The trading desk or back office must keep detailed tables for each

EXHIBIT 6
Prospectus-Based Cota Calculations for Omni V

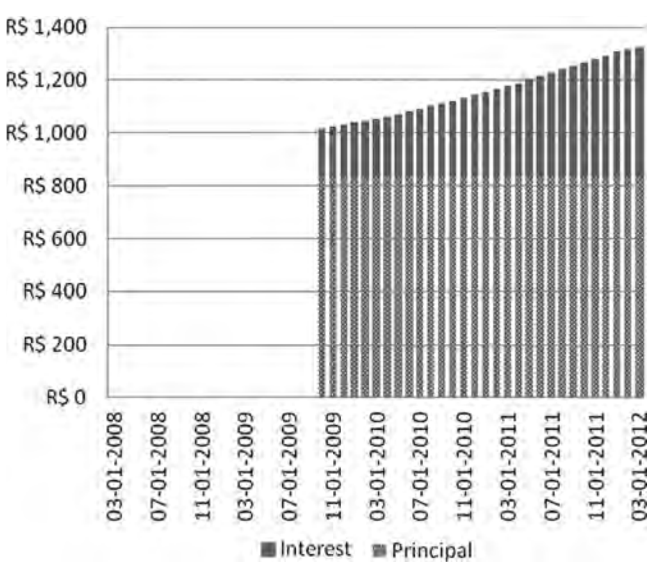
Date	Administrator Price	DI Rate	CETIP Daily Factor	Prospectus DI Rate in Daily Terms	Prospectus Factor DI	Prospectus VC(se)	Difference Administrator Price and VC(se)
03-14-2008	25,000.00	11.10	1.000417790	1.0004177877	1.0004679200	25,000.00	0.00
03-17-2008	25,011.70	11.09	1.000417430	1.0004174303	1.0004675200	25,011.70	0.00
03-18-2008	25,023.39	11.09	1.000417430	1.0004174303	1.0004675200	25,023.39	0.00
03-19-2008	25,035.09	11.10	1.000417790	1.0004177877	1.0004679200	25,035.09	0.00
03-20-2008	25,046.81	11.11	1.000418140	1.0004181450	1.0004683200	25,046.80	–0.01
03-24-2008	25,058.54	11.10	1.000417790	1.0004177877	1.0004679200	25,058.53	–0.01
03-25-2008	25,070.26	11.10	1.000417790	1.0004177877	1.0004679200	25,070.26	0.00
03-26-2008	25,081.99	11.09	1.000417430	1.0004174303	1.0004675200	25,081.99	0.00
03-27-2008	25,093.72	11.09	1.000417430	1.0004174303	1.0004675200	25,093.72	0.00
03-28-2008	25,105.45	11.09	1.000417430	1.0004174303	1.0004675200	25,105.45	0.00
03-31-2008	25,117.19	11.15	1.000419570	1.0004195739	1.0004699200	25,117.19	0.00
04-01-2008	25,128.99	11.14	1.000419220	1.0004192167	1.0004695200	25,128.99	0.00

Sources: CETIP, CVM, Uqbar's Orbis System (daily administrator prices).

EXHIBIT 7
Omni V Cota Price Calculation with Amortization

Work Days	Date	Impl. Forward CDI	One Day Int. Factor	Cum. Int. Factor	Cumulative Price	Interest Payment	Principal Amort.	Cum. Price–Int.–Prin.
1	12/31/2010	0.106398	1.000449	1.000449	17155.839	0.000	0.000	17155.839
2	1/3/2011	0.10652	1.00045	1.0009	17163.558	0.000	0.000	17163.558
3	1/4/2011	0.106702	1.000451	1.001351	17171.294	0.000	0.000	17171.294
4	1/5/2011	0.10688	1.000451	1.001803	17179.045	0.000	0.000	17179.045
5	1/6/2011	0.107041	1.000452	1.002256	17186.811	0.000	0.000	17186.811
6	1/7/2011	0.107202	1.000453	1.002709	17194.591	0.000	0.000	17194.591
7	1/10/2011	0.107368	1.000453	1.003164	17202.386	0.000	0.000	17202.386
8	1/11/2011	0.107532	1.000454	1.003619	17210.197	0.000	0.000	17210.197
9	1/12/2011	0.107695	1.000455	1.004076	17218.022	0.000	0.000	17218.022
10	1/13/2011	0.107857	1.000455	1.004533	17225.862	0.000	0.000	17225.862
11	1/14/2011	0.108019	1.000456	1.004991	17233.716	0.000	0.000	17233.716
12	1/17/2011	0.10818	1.000457	1.00545	17241.585	0.000	0.000	17241.585
13	1/18/2011	0.108341	1.000457	1.00591	17249.470	0.000	0.000	17249.470
14	1/19/2011	0.108502	1.000458	1.00637	17257.368	0.000	0.000	17257.368
15	1/20/2011	0.108661	1.000459	1.006832	17265.282	0.000	0.000	17265.282
16	1/21/2011	0.108821	1.000459	1.007294	17273.210	0.000	0.000	17273.210
17	1/24/2011	0.108979	1.00046	1.007757	17281.152	0.000	0.000	17281.152
18	1/25/2011	0.109137	1.00046	1.008221	17289.110	0.000	0.000	17289.110
19	1/26/2011	0.109295	1.000461	1.008686	17297.081	0.000	0.000	17297.081
20	1/27/2011	0.109452	1.000462	1.009152	17305.068	0.000	0.000	17305.068
21	1/28/2011	0.109608	1.000462	1.009618	17313.069	0.000	0.000	17313.069
22	1/31/2011	0.109764	1.000463	1.010086	17321.084	321.406	833.333	16166.345

EXHIBIT 8
Omni V Payoff Profile



FIDC with the ability to load historic factors and apply the correct rounding and truncation as specified in the prospectus. In addition, the firm must keep amortization tables or have applications that calculate the tables quickly.

MARKING FIDCs TO MARKET

As stated earlier, cota values provided by administrators are accounting entries. They can be considered to be book-value prices if the analyst wants to constrain the net present value calculation to use the assumptions at issue. They are useful as a reference but generally do not provide the correct value for trading.

FIDC traders need a methodology for incorporating past deal performance, current market conditions, and expectations for the market in the future. The Brazilian market has developed simplified methods for calculating prices that add a spread to an underlying index for risk-free yields. Many measures of spread, such as spread to weighted-average life (WAL), select one point on the curve to use as a representative yield. Swap-based technology weights each cash-flow contribution to the yield of the security. This provides a much more realistic yield for benchmarking or measuring the yield of an ABS security. The next section demonstrates various yield and spread measures used to price and evaluate FIDCs.

Trader Spread

Brazilian investors use a back-of-the-envelope calculation to compare a spread-plus FIDC (Chemical V) to a percent of DI issue (Omni V). They simply multiply the percentage factor multiplier by the current CDI rate and then subtract the CDI rate.

$$\text{Trader spread} = (\text{FM} \times \text{CDI}) - \text{CDI}$$
$$\text{Trader spread}_{12/31/2010} = (112\% \times 10.64\%) - 10.64\% = 1.28\%$$

Trader spread changes with the level of interest rates, however, and provides an inaccurate assessment of expected future trader spreads. In Exhibit 9, we see that on December 31, 2010, the trader spread is 1.28% for Omni V, but the average expected future trader spread based on the forward-forward rates is 1.60%. Clearly the trader spread underestimates the probable realized returns when the yield curve is positively sloped and vice versa. We will see more examples of the very misleading nature of this indicator of spread component in other exhibits.

IRR Spread—Cota IRR versus Government IRR

Once we have the accurate future cash flows based on correct coupon calculations and forward rates, we can use the internal rate of return (IRR) to discount the cash flows and calculate their present value. This provides the yield at which we could theoretically swap the floating-rate payments for fixed with a swap. To provide a reference for this yield, we generate the coupon payments without the spread to determine the yield for a pure swap. We then adjust the IRR for both to change monthly (30/360) to the bus/252 day count convention based on the instructions from CETIP (see CETIP [2005]). We call the difference the IRR spread.

Z-Spread—Pricing with the Zero-Volatility Curve

The best-practice approach to pricing ABS securities starts with the arbitrage-free technology for pricing swaps. First, the system uses CDI contract spot prices observed in the market and curve fitting technology, such as a cubic spline to estimate the current interest rate curve as a continuous function of interest rates (see McCulloch [1975]). This function is then used to estimate the forward-forward rates; that is the forward-looking periodic series of rates that matches the frequency for resetting the index. In the case of Brazil, we estimate a forward-forward series of daily rates that is consistent with today's spot curve.

We use the forward-forward rates adjusted for the spread to generate the future coupons for the deal. Then we use the spot rates from the curve function to determine the rate for the specific day of the cash flow to discount the interest and/or amortization to obtain the present value. The procedure then adds a constant market spread to each risk-free spot rate to arrive at a yield curve that discounts the cash flows back to the observed price. The spread reflects the risk perceived by the market for the deal. This curve is called the zero-volatility curve because it reflects market perceptions of curve evolution with 100% probability or zero volatility. The constant spread that we add to the CDI curve is called the Z-spread.

There is much debate over which curve-fitting process is best, and an enormous amount of literature explains swap-based technology for pricing ABS

EXHIBIT 9

Omni V Implied Future Trader Spreads based on Forward-Forward Rates, December 30, 2010

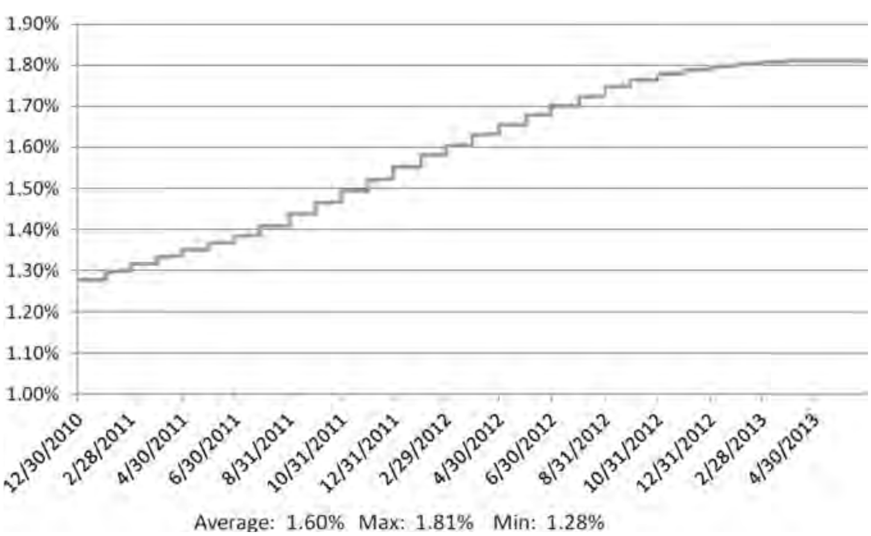


EXHIBIT 10
Recombining Curve

Observed Rates			Recombined Rates		
Future Dates as of 12/31/2010	Spot Curve	Forward-Forward Curve	Recombining Factor	Recombined Rate	Difference between Spot Rate and Recombined Rate
12/31/2010	10.64%	10.64%	1.000401	0.1064	0.0000%
1/3/2011	10.65%	10.65%	1.000803	0.1065	0.0000%
1/4/2011	10.65%	10.67%	1.001206	0.1065	0.0000%
1/5/2011	10.66%	10.69%	1.001609	0.1066	0.0000%
1/6/2011	10.67%	10.70%	1.002014	0.1067	0.0000%
1/7/2011	10.68%	10.72%	1.002419	0.1068	0.0000%
1/10/2011	10.69%	10.74%	1.002825	0.1069	0.0000%
1/11/2011	10.70%	10.75%	1.003231	0.1070	0.0000%
1/12/2011	10.70%	10.77%	1.003638	0.1070	0.0000%
1/13/2011	10.71%	10.79%	1.004046	0.1071	0.0000%
1/14/2011	10.72%	10.80%	1.004455	0.1072	0.0000%
1/17/2011	10.73%	10.82%	1.004865	0.1073	0.0000%
1/18/2011	10.74%	10.83%	1.005275	0.1074	0.0000%
1/19/2011	10.74%	10.85%	1.005686	0.1074	0.0000%
1/20/2011	10.75%	10.87%	1.006098	0.1075	0.0000%
1/21/2011	10.76%	10.88%	1.00651	0.1076	0.0000%
1/24/2011	10.77%	10.90%	1.006923	0.1077	0.0000%
1/25/2011	10.78%	10.91%	1.007337	0.1078	0.0000%
1/26/2011	10.78%	10.93%	1.007752	0.1078	0.0000%
1/27/2011	10.79%	10.95%	1.008167	0.1079	0.0000%
1/28/2011	10.80%	10.96%	1.008584	0.1080	0.0000%
1/31/2011	10.81%	10.98%	1.009001	0.1081	0.0000%

securities. We are not going to address these issues here. We refer the reader to Adams and Van Deventer [1994], Hull and White [1993], and the other research cited in the References at the end of this article for more information. We prefer the cubic spline method and found that this generally works well for the Brazilian market. What is important is that the process generates a smooth, continuous curve of spot rates and forward-forward rates. Furthermore, the forward-forward rates must recombine to the observed spot rates. See Exhibit 10.

As we saw in Exhibits 5 and 8, the securities have completely different payoff profiles; Chemical V almost resembles a regular bond, whereas the Omni V payoff looks like a Z CMO. Moreover, Omni V has already started to amortize on the pricing date. Swap-curve technology enables investors to overcome differences in cash flows and estimate consistent measures of value for a variety of cash flow streams. (See CETIP [2005]) for the precise calculations.)

OPTION-ADJUSTED SPREAD TECHNOLOGY

Although neither bond has option features, the analytics for path-dependent securities provide interesting insights into the returns for Omni V. Path-dependent solutions to evaluating bonds with and without options have been widely researched, widely published, and used with many different approaches. We refer the reader to Ho [1992].

We chose the linear path space (LPS) for generating our interest rate paths (Ho [1992]). The LPS uses a trinomial approach to sample the Ho–Lee binomial lattice from Ho and Lee [1986]. We adapted the LPS to the Brazilian market, and it creates 200 arbitrage-free interest rate paths with daily forward-forward rates. The LPS employs a short-rate volatility of 20% annually and a long-rate volatility of 10%. This approach to mean reversion prevents the model from generating negative interest rates. We estimate short-rate volatility and long-rate volatility outside of the model. Similar curve-generation models and Monte Carlo simulations are available through MatLab directly or as MatLab freeware.

The LPS approach has an advantage relative to Monte Carlo simulation in that it needs only about 200 to 300 paths, whereas Monte Carlo simulation often involves thousands of paths. LPS assigns a probability to each path, and this allows us to use fewer paths to run the simulation on the two FIDCs.

Exhibit 11

IRR Spread vs. Path Probability from Linear Path Space

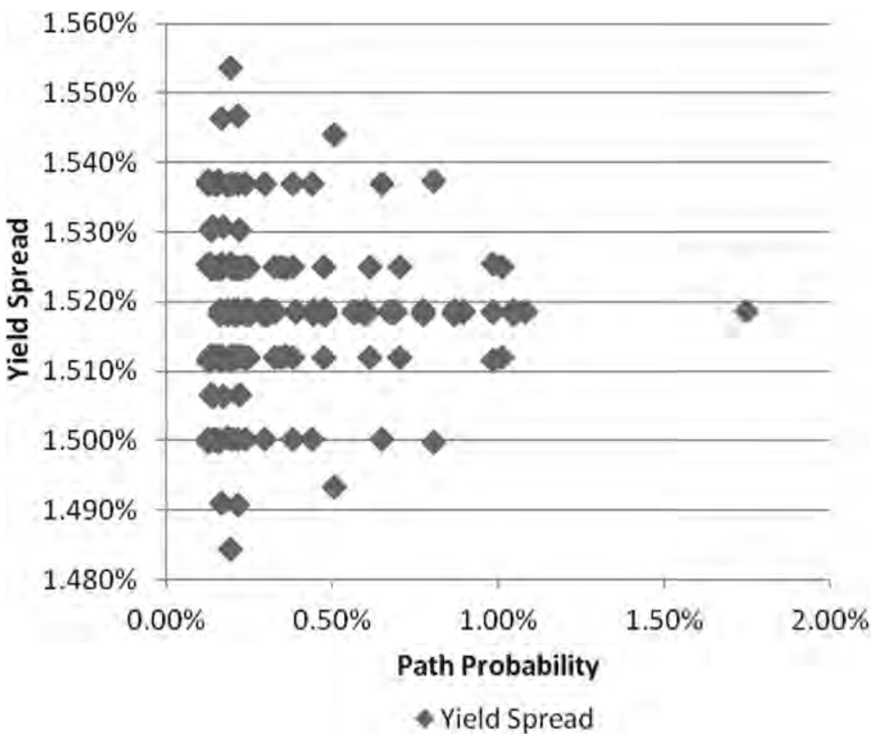
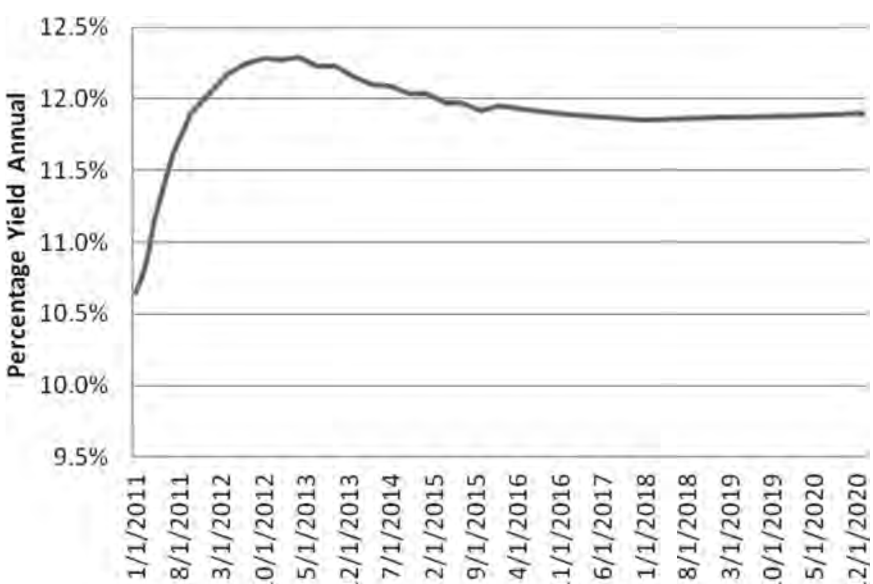


Exhibit 12

Brazilian Swap Curve—Spot Yields, December 30, 2010



We replicate the Z-spread process for each path; that is, we set up the LPS with a given curve and the volatility from recent history and then use the forward-forward CDI rates to generate the coupons and the curve implied by the scenario to discount the cash flows. The goal is to find the one spread (option-adjusted spread or OAS) for all paths that gives an average cota value equal to the current cota value. We can see in Exhibit 11 that the distribution of the yield spreads is normal and symmetrical (the FIDC IRR for each LPS scenario minus the swap IRR for that same scenario). See also Exhibit 12.

Pricing Results

We priced both issues on December 30, 2012, and again on September 30, 2011, to demonstrate the effect of the shape of the yield curve in making an investment decision. We also employed a host of yield and spread measures to demonstrate how misleading some market practices can be. See Exhibits 13, 14, and 15. The curve was upward sloping on December 30, 2012, but negatively sloped on September 30, 2011.

First, the spread for Chemical V is constant, except for the IRR spread, because adding 125 basis points to the daily CDI rate is the definition of the OAS and the Z-spread. However, all of the spreads for Omni depend on the current CDI rate and the shape of the curve. The Z-spread and the OAS for a percent-of-DI coupon FIDC will always be greater than the trader spread in a market with a positive slope curve market and less than the trader spread in a negative slope curve market. The Z-spread is a more dependable measure of expected returns because it measures market expectations for the entire curve.

EXHIBIT 13

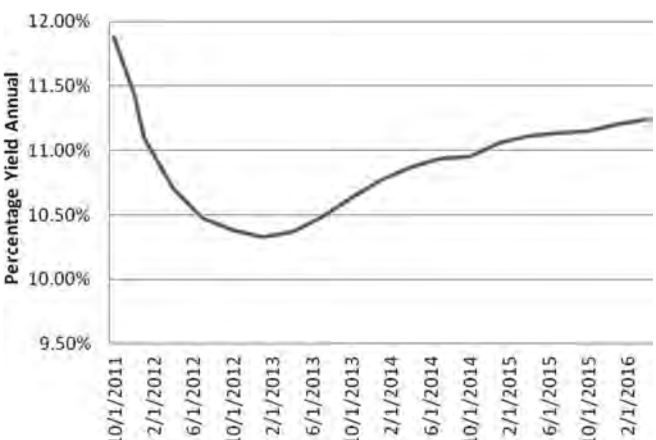
Comparison of Spread Measures, December 30, 2010

Measure	Chemical V	Omni Financeira V
Original Cota Value	R$1,000.00	R$25,000.00
Factor	1.0000	0.5000
Cota Value (12/30/2010)	R$1,000.00	R$17,148.13
Accrued Interest	R$ 0.00	R$4,548.13
Spread	125 bps	112% multiplier
Trader Spread	125 bps	128 bps
IRR Spread*	140 bps	153 bps
Z-Spread	125 bps	144 bps
OAS	125 bps	136 bps

**Note: If we use the typical non-multiplicative spread formula that is standard in the U.S. or Europe, the IRR spread is 125 bps for Chemical V.*

EXHIBIT 14

Brazilian Swap Curve—Spot Yields, September 30, 2011



Source: CETIP.

EXHIBIT 15

Comparison of Spread Measures on September 30, 2011

Measure	Chemical V	Omni Financeira V
Original Cota Value	R$1,000.00	R$25,000.00
Factor	1.0000	0.2000
Cota Value (9/30/2011)	R$1,000.00	R$7,533.70
Accrued Interest	R$0.00	R$2,533.70
Spread	125 bps	112% multiplier
Trader Spread	125 bps	142 bps
IRR Spread	137 bps	136 bps
Z-Spread	125 bps	125 bps
OAS	125 bps	125 bps

When comparing returns on the two issues, the OAS provides the most reasonable estimate of spread for the long-term scenario because it incorporates information about all rate paths in the simulation. We know this is true for Omni V because it matured in March 2012 and we can estimate realized IRR spread versus the expected IRR spread that is consistent with an OAS of 136 basis points—the senior cotas realized 148 bps versus 153 bps for the average IRR spread from all the outcomes from the LPS simulation.

CONCLUSION AND RECOMMENDATIONS

This article shows the usefulness of best practices for calculating prices and spreads for Brazilian FIDCs. We show that advanced analytics have the potential to aid investors to understand the economic payoffs when evaluating FIDCs. Swap-based pricing was developed because of the deficiencies in older technology and the arbitrage-free, swap-based approach better demonstrates the likely payoffs. Basic coupon calculations can err dramatically in estimating the excess spread earned over the life of a deal. We find that swap-based analytics and path-wise analytics provide the best solution for the more-complicated FIDCs.

We recommend that CETIP and the Associação das Entidades dos Mercados Financeiro e de Capitais (ANBIMA) work with the CVM to set standard calculation assumptions for each methodology for calculating the coupons and amortization. This means standards for rounding, truncating, and calculating spreads. These agencies should then work with the administrators, custodians, and issuers to ensure that these standards are adopted across the entire industry.

The CVM should also stipulate that administrators have the technology to calculate and post basic interest calculations and current face value or the factor for the deal on a daily basis.

In addition, the CVM could promote a portal to disseminate the data more efficiently and provide easy access to basic trading information. The organization and standardization of data would provide uniform information for market participants, and developers and traders would find it easier to use the data in their applications for pricing and analysis.

REFERENCES

Adams, K.J., and D.R. Van Deventer. "Fitting Curves and Forward Rate Curves with Maximum Smoothness." *The Journal of Fixed Income*, Vol. 4, No. 1 (June 1994), pp. 52-62.

Budinger, V.H. "Analyzing Brazilian ABS: FIDCs." *The Journal of Structured Finance*, Vol. 18, No. 1 (Spring 2012), pp. 100-122.

CETIP. "Caderno de Fórmulas." Website, April 2005: http://www.cetip.com.br/Comunicados-Documentos/UnidadeTitulos/.

Ho, T.S.Y. "Managing Illiquid Bonds and Linear Path Space." *The Journal of Fixed Income*, Vol. 2, No. 1 (June 1992), pp. 80-94.

Ho, T.S.Y., and S.-B. Lee. "Term Structure Movements and Pricing Interest Rate Contingent Claims." *Journal of Finance*, Vol. 41, No. 5 (December 1986), pp. 1011-1029.

Hull, J., and A. White. "One-Factor Interest-Rate Models and the Valuation of Interest Rate Derivative Securities." *Journal of Financial and Quantitative Analysis*, 28 (1993), pp. 235-254.

McCulloch, J. "The Tax-Adjusted Yield Curve." *Journal of Finance*, 30 (1975), pp. 811-830.